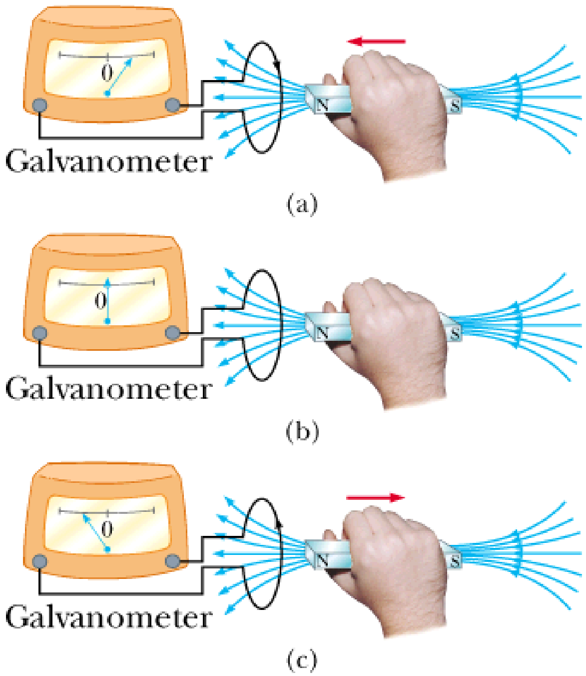
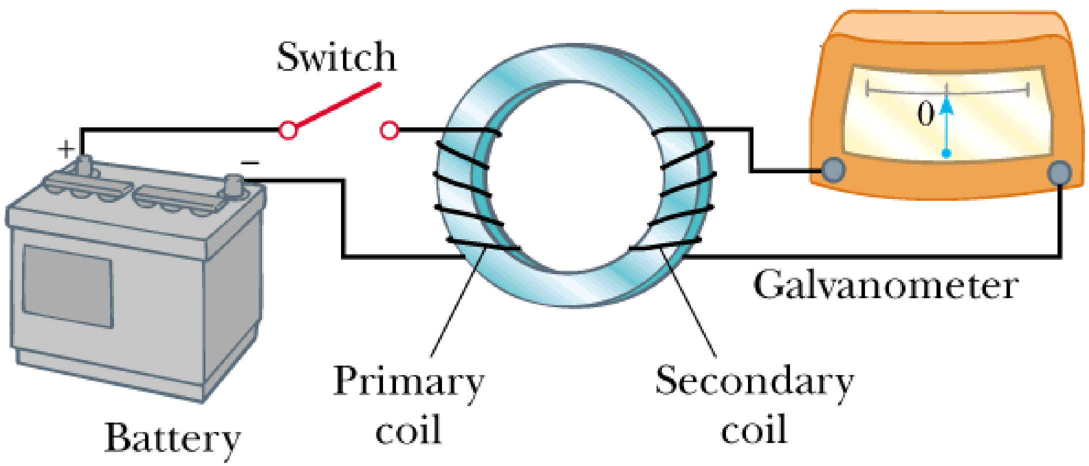
Faraday's Law

- Faraday's Law of Induction
- Motional emf
  - Motors and Generators
- Lenz's Law
  - Eddy Currents

Induced EMF



A current flows through the loop when a magnet is moved near it, without any batteries!



The needle deflects momentarily when the switch is closed

Faraday's Law of Induction

The emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit.

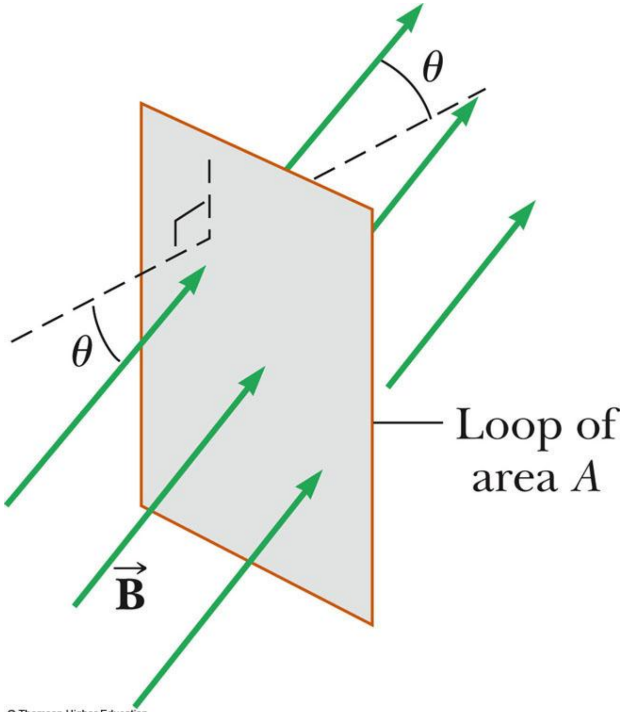
$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

where, $\Phi_B = \int \vec{\mathbf{B}}.d\vec{\mathbf{A}}$

For N loops, $\boxed{\mathcal{E} = -N\frac{d\Phi_B}{dt}}$

Faraday's Law of Induction

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \mathcal{E} = -\frac{d}{dt}\left(BA\cos\theta\right)$$

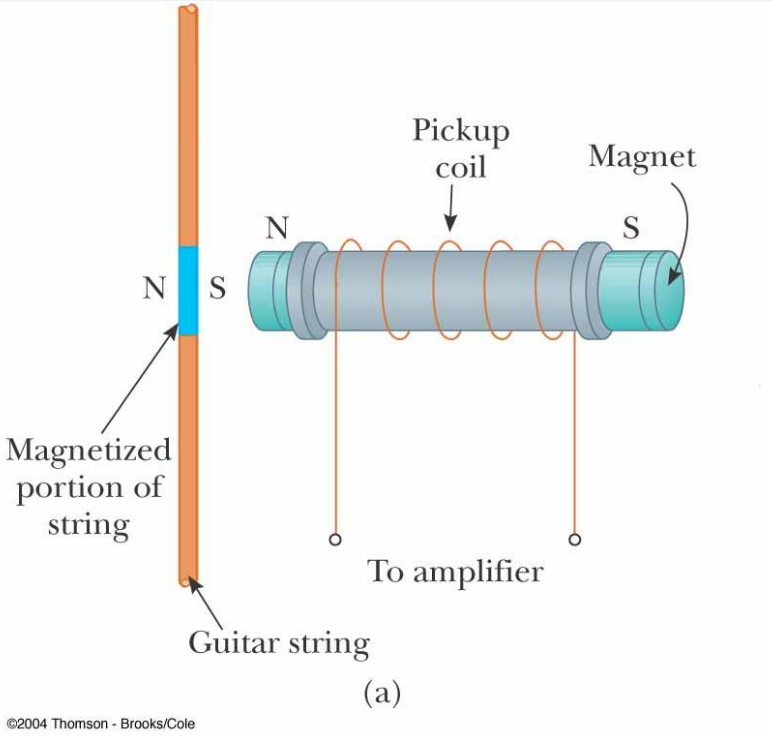


To induce an emf we can change,

- the magnitude of B
- the area enclosed by the loop
- the angle between B and the normal to the area
- any combination of the above

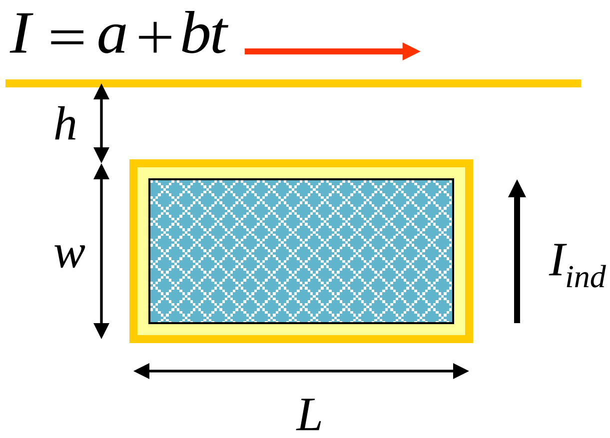
over time.

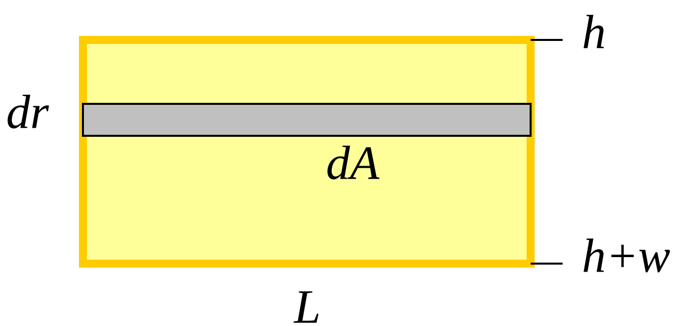
Electric Guitar



- The permanent magnet inside the coil magnetizes the string.
- When the string vibrates, it creates a time varying magnetic flux through the coil and induces an emf in it.
- The induced emf is then amplified and fed to speakers.

Problem 31.8





$$A = Lw$$

$$dA = Ldr$$

$$B = \frac{\mu_0 I}{2\pi r} \qquad \Phi_B = \int \vec{\mathbf{B}}.d\vec{\mathbf{A}}$$

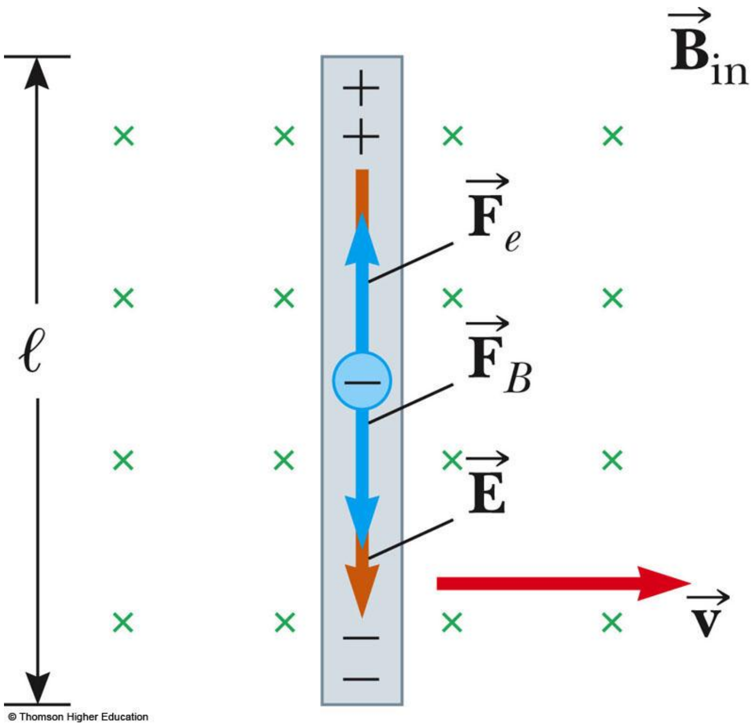
$$\Phi_B = \int \vec{\mathbf{B}}.d\vec{\mathbf{A}} = \int_h^{h+w} \frac{\mu_0 I}{2\pi r} L dr$$

$$\Phi_B = \frac{\mu_0 I L}{2\pi} \ln\left(\frac{h+w}{h}\right)$$

$$\Phi_B(t) = \frac{\mu_0 (a+bt) L}{2\pi} \ln\left(\frac{h+w}{h}\right)$$

$$\mathcal{E} = -\frac{d\Phi_B(t)}{dt} = -\frac{\mu_0 b L}{2\pi} \ln\left(\frac{h+w}{h}\right)$$

Motional EMF



As the wire moves,

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Which sets the charges in motion in the direction of F_B and leaves positive charges behind.

As they accumulate on the bottom, an electric field is set up inside.

In equilibrium,

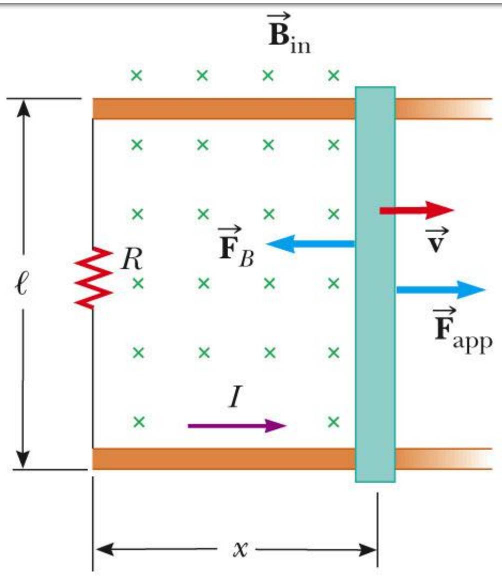
$$F_B = F_e$$

$$qvB = qE$$

$$\Delta V = El = Blv$$

$$E = vB$$

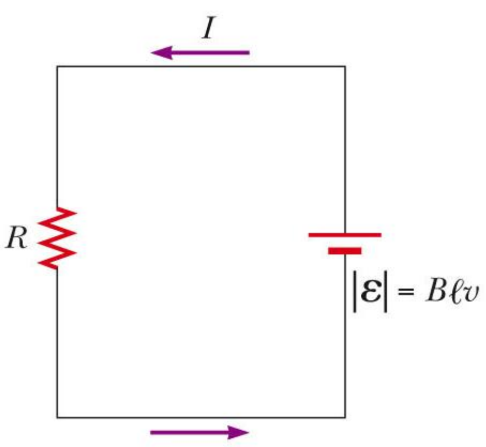
Motional EMF in a Circuit



$$\Phi_B = BA = Blx$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(Blx\right) = -Bl\,\frac{dx}{dt}$$

$$\mathcal{E} = -Blv \longrightarrow I = \frac{|\mathcal{E}|}{R} = \frac{Blv}{R}$$

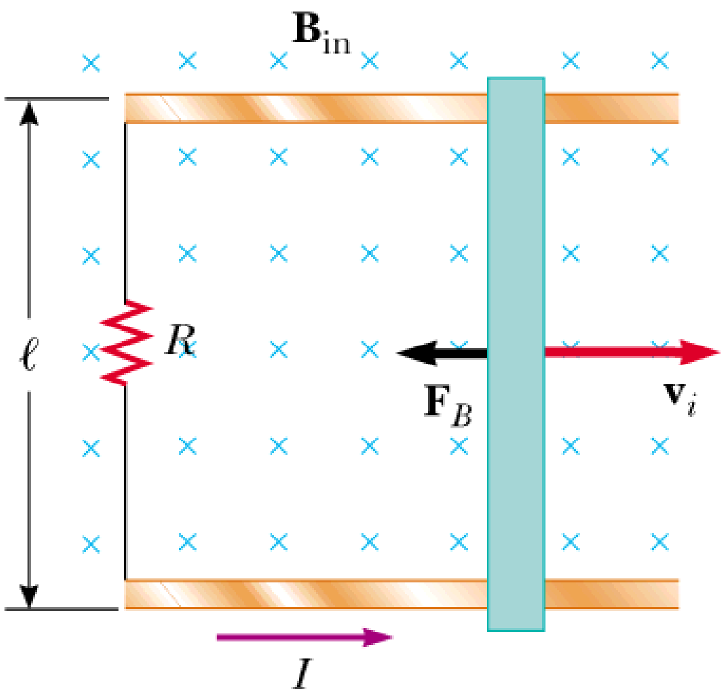


If the bar is moved with constant velocity,

$$F_{app} = F_B = IlB$$

$$\mathcal{P} = F_{app}v = \left(IlB\right)v = \frac{B^2l^2v^2}{R} = \frac{\mathcal{E}^2}{R}$$

Magnetic Force on a Sliding Bar



The bar has a mass, *m*, and an initial velocity $\boldsymbol{v}_i$

$$F_B = -IlB$$

$$F_x = ma = m\frac{dv}{dt} = -IlB$$

$$I = \frac{Blv}{R}$$

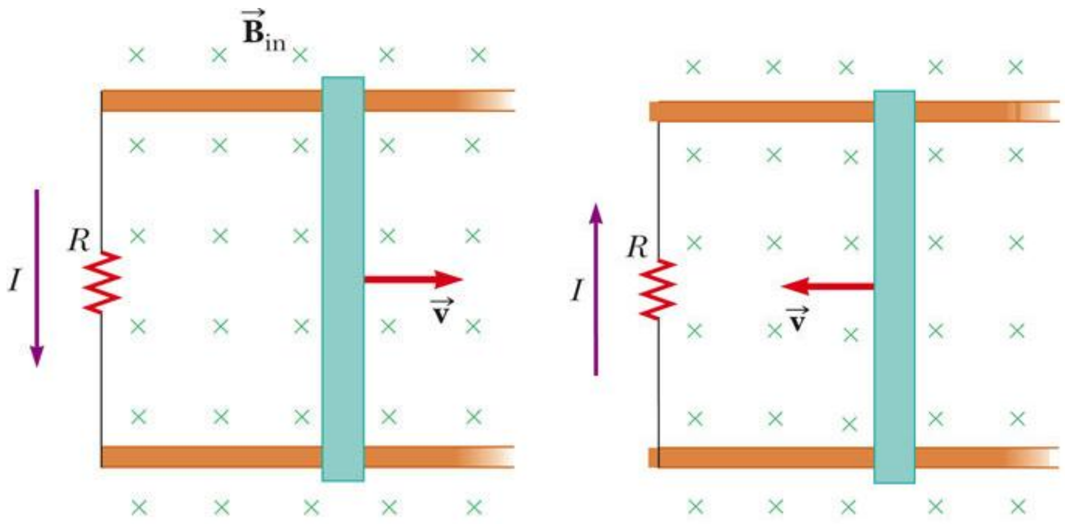
$$m\frac{dv}{dt} = -\frac{B^2l^2}{R}v$$

$$\frac{dv}{v} = -\left(\frac{B^2l^2}{mR}\right)dt$$

$$\boxed{v = v_i e^{-t/\tau}} \qquad \tau = \left(\frac{B^2l^2}{mR}\right)^{-1}$$

Lenz's Law

The polarity of the induced emf is such that it tends to produce a current that creates a magnetic flux to oppose the change in magnetic flux through the area enclosed by the current loop.



As the bar is slid to the right, the flux through the loop increases.

This induces an emf that will result in an opposing flux.

Since the external field is into the screen, the induced field has to be out of the screen.

Which means a counterclockwise current

Energy Considerations

Suppose, instead of flowing counterclockwise, the induced current flows clockwise:

Then the force will be towards the right

which will accelerate the bar to the right

which will increase the magnetic flux

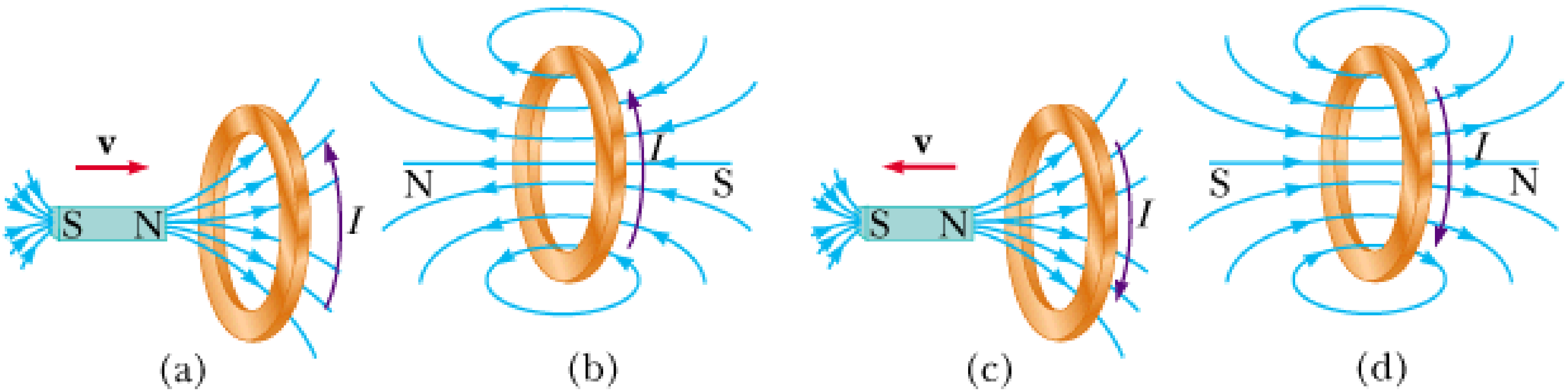
which will cause more induced current to flow

which will increase the force on the bar

… and so on

All this is inconsistent with the conservation of energy

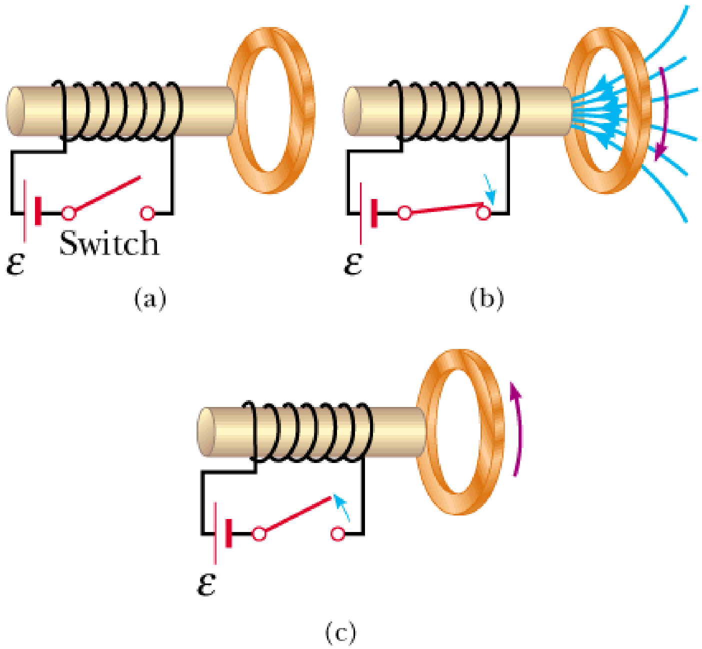
Moving Magnet and Stationary Coil



- Right moving magnet increases flux through the loop.
- It induces a current that creates it own magnetic field to oppose the flux increase.

- Left moving magnet decreases flux through the loop.
- It induces a current that creates it own magnetic field to oppose the flux decrease.

Application of Lenz's Law



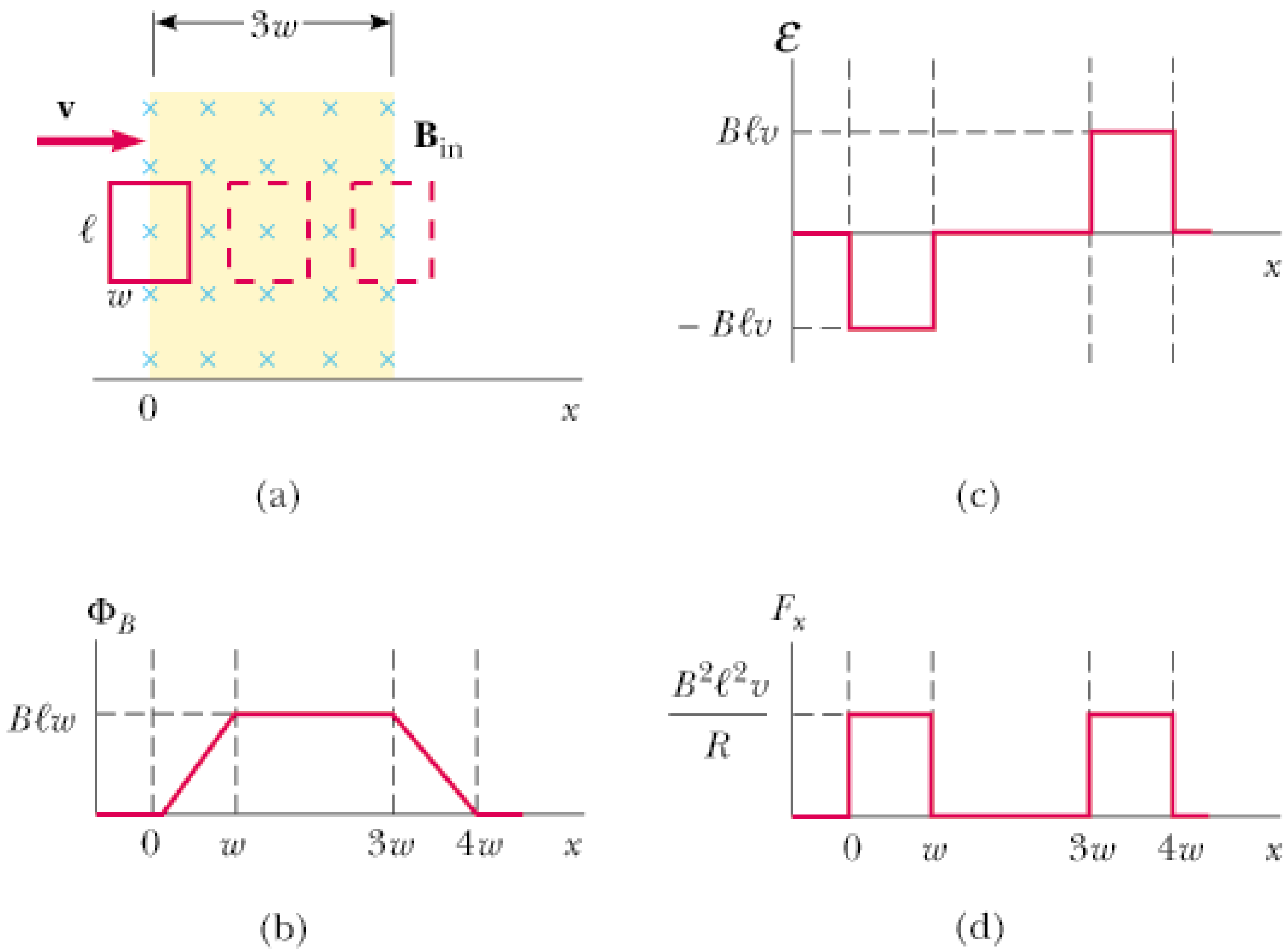
When the switch is closed, the flux goes from zero to a finite value in the direction shown.

To counteract this flux, the induced current in the ring has to create a field in the opposite direction.

After a few seconds, since there is no change in the flux, no current flows.

When the switch is opened again, this time flux decreases, so a current in the opposite direction will be induced to counter act this decrease.

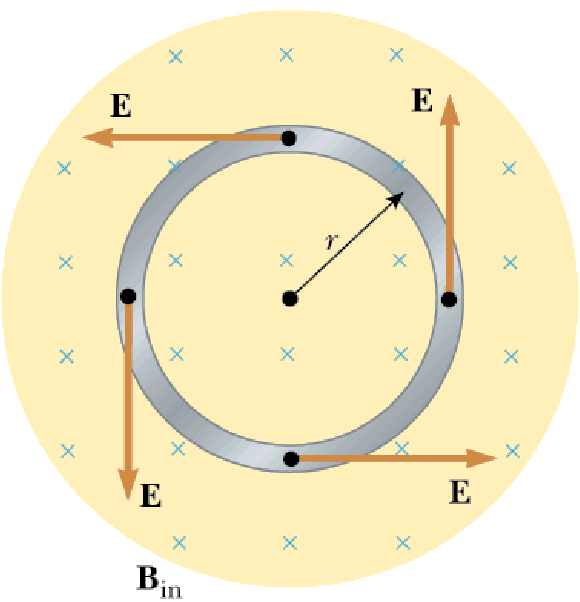
Loop Moving Through a Magnetic Field



Induced EMF and Electric Fields

Changing Magnetic Flux → EMF → **Electric Field Inside a Conductor**

This induced electric field is non-conservative and time-varying



$$\mathcal{E} = -\frac{d\Phi_B}{dt} \longrightarrow \oint \vec{\mathbf{E}}.d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

General Form of Faraday's Law

$$W = q\mathcal{E} = F_E(2\pi r)$$

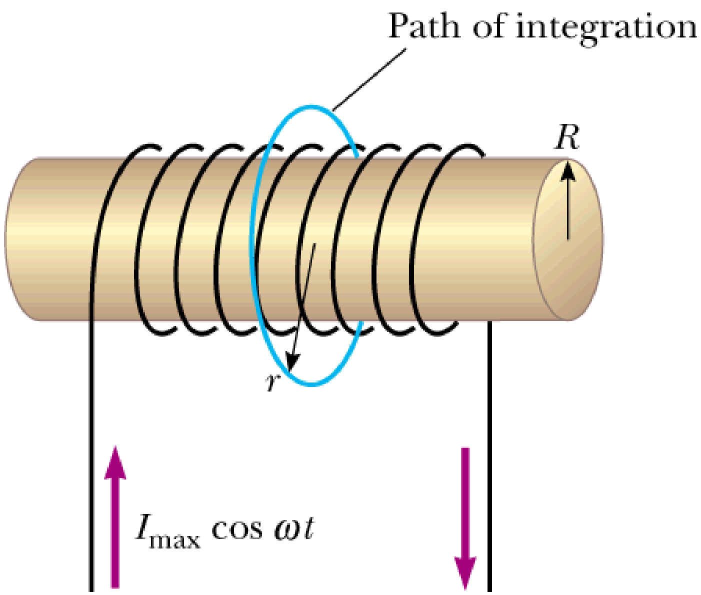
$$q\mathcal{E} = qE(2\pi r)$$

$$E = \frac{\mathcal{E}}{2\pi r}$$

$$E = -\frac{1}{2\pi r}\frac{d\Phi_B}{dt} = -\frac{1}{2\pi r}\frac{d}{dt}\left(\pi r^2 B\right)$$

$$E = -\frac{r}{2}\frac{dB}{dt}$$

Electric Field Induced by a Changing Magnetic Field in a Solenoid



$$I(t) = I_{max} \cos \omega t$$

$$B = \mu_0 n I$$

$$B(t) = \mu_0 n I_{max} \cos \omega t$$

$$\oint \mathbf{E}.d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{E}.d\mathbf{s} = -\frac{d}{dt}\left(B\pi R^2\right) = -\pi R^2 \frac{dB}{dt}$$

$$E(2\pi r) = -\pi R^2 \frac{dB}{dt}$$

$$E(2\pi r) = \pi R^2 \mu_0 n I_{max} \omega \sin \omega t$$

$$\boxed{E = \frac{\mu_0 n I_{max} \omega R^2}{2r} \sin \omega t} \qquad r > R$$

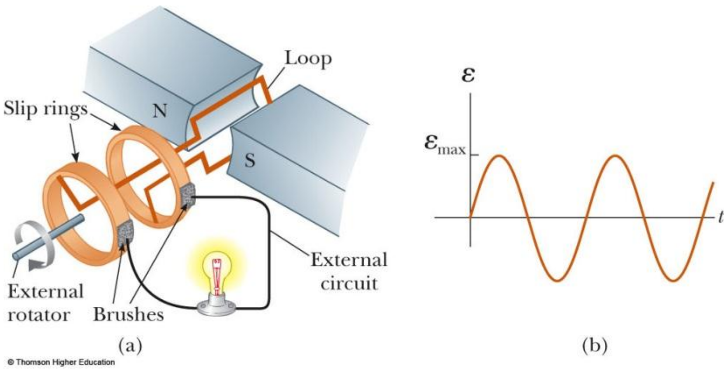
$$\Phi_B = B\pi r^2$$

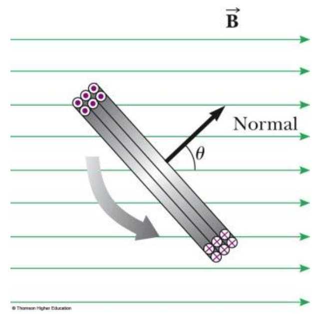
$$E(2\pi r) = -\pi r^2 \frac{dB}{dt} = \pi r^2 \mu_0 n I_{max} \omega \sin \omega t$$

$$\boxed{E = \frac{\mu_0 n I_{max} \omega}{2} r \sin \omega t} \qquad r < R$$

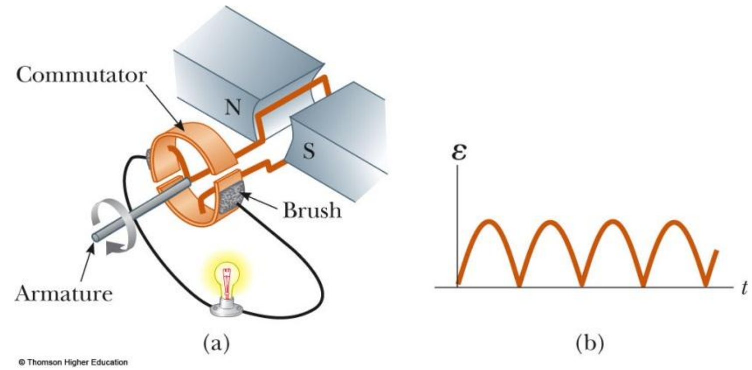
Generators and Motors

AC Generator





DC Generator

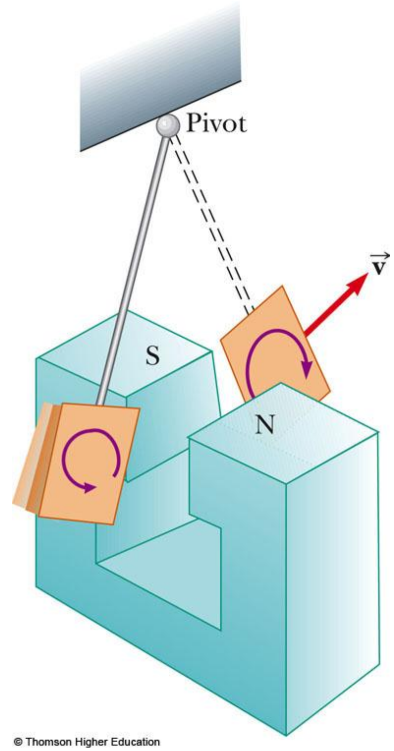


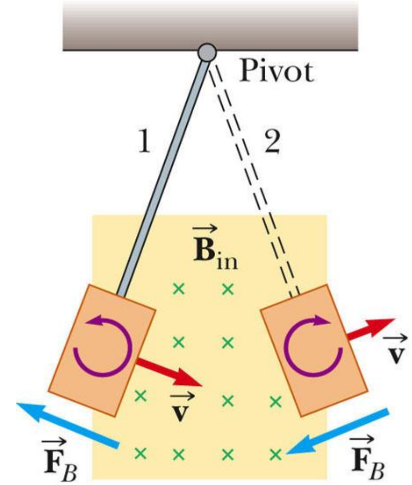
$$\Phi_B = BA\cos\theta = BA\cos\omega t \longrightarrow \begin{aligned} \mathcal{E} &= -N\frac{d\Phi_B}{dt} = -NAB\frac{d}{dt}\left(\cos\omega t\right) = NAB\omega\sin\omega t \\ \mathcal{E}_{\max} &= NAB\omega \end{aligned}$$

A motor is a generator in reverse. A time-varying current is applied to the loop. The resulting torque rotates the loop and the shaft connected to it.

Eddy Currents

- If a solid piece of conductor moves in and out of a magnetic field, circulating currents can be induced in the conductor.
- These currents rise from induction and obey Lenz's Law.
- Since they result in undesirable energy transfer to internal energy, they are generally minimized by layering or sectioning large metal components.





For Next Class

- Reading Assignment
  - Chapter 32 – Inductance
- WebAssign: Assignment 9